Solutions for Exam 2–Morning Section

1. $$\mathcal{L}\{hu(t)\} = h\mathcal{L}\{u(t)\} = h\left(\frac{1}{s}\right) = h/s$$

Answer is D.

2. $$m = \left.\frac{dy}{dx}\right|_{x=5} = 24x = (24)(5)$$
$$= 120$$

Answer is B.

3. $y = 6 - 2x$ is the equation of the line.
$y \le 6 - 2x$ gives the area shown.

Answer is B.

4. Use the substitution method.

$$x = \frac{7}{3} + 2y$$
$$(2)\left(\frac{7}{3} + 2y\right) - 11y = -5$$
$$y = \frac{29}{21}$$
$$x = \frac{7}{3} + (2)\left(\frac{29}{21}\right) = \frac{107}{21}$$

Answer is A.

5. $$V = \mathbf{V}_1 \cdot (\mathbf{V}_2 \times \mathbf{V}_3)$$
$$\mathbf{V}_2 \times \mathbf{V}_3 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 1 & -2 \\ 0 & 1 & 2 \end{vmatrix} = 4\mathbf{i} + 2\mathbf{j} - \mathbf{k}$$
$$\mathbf{V}_1 \cdot (\mathbf{V}_2 \times \mathbf{V}_3) = 12 + 4 - 1 = 15$$

Answer is C.

6. $$(\cot^2\theta)(\sin^2\theta) + \frac{1}{\csc^2\theta} = \left(\frac{\cos^2\theta}{\sin^2\theta}\right)(\sin^2\theta) + \sin^2\theta$$
$$= \cos^2\theta + \sin^2\theta$$
$$= 1$$

Answer is D.

7. $$\frac{d\sqrt{2x + 9x^2}}{dx} = \left(\tfrac{1}{2}\right)\left(\frac{1}{\sqrt{2x + 9x^2}}\right)(2 + 18x)$$
$$= \frac{1 + 9x}{\sqrt{2x + 9x^2}}$$

Answer is A.

8. Use L'Hôpital's rule.

$$\frac{\frac{d}{dx}(10x^2 - 5x + 1)}{\frac{d}{dx}(10x^2 - 6x)} = \frac{20x - 5}{20x - 6}$$

Use L'Hôpital's rule again.

$$\frac{\frac{d}{dx}(20x - 5)}{\frac{d}{dx}(20x - 6)} = \frac{20}{20} = 1$$

Answer is D.

9. $$\int x(x + 1)dx = \int (x^2 + x)dx$$
$$= \frac{x^3}{3} + \frac{x^2}{2} + C$$

Answer is C.

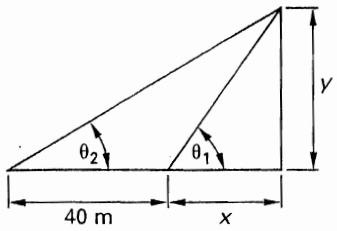
10. Leading zeros are not significant, so the number has 10 significant digits.

Answer is C.

11. $$\overline{x} = \frac{\sum x_i}{n} = \frac{30.46}{7} = 4.351$$
$$s = \sqrt{\frac{\sum (x_i - \overline{x})^2}{n - 1}} = \sqrt{\frac{0.005887}{7 - 1}} = 0.03132$$

Answer is D.

12.



$$\tan\theta_2 = \frac{y}{x + 40\text{ m}} = \frac{x\tan\theta_1}{x + 40\text{ m}}$$

$$(x + 40\text{ m})(\tan\theta_2) = x\tan\theta_1$$

$$\theta_1 = 46.8°$$

$$\theta_2 = 29.23°$$

$$x = \frac{(-40\text{ m})(\tan\theta_2)}{\tan\theta_2 - \tan\theta_1} = 46.4\text{ m}$$

Answer is B.

13. $$\cos\theta = \frac{\mathbf{V}_1 \cdot \mathbf{V}_3}{|\mathbf{V}_1||\mathbf{V}_3|}$$

$$= \frac{(3)(2) + (2)(3) + (1)(2)}{\sqrt{(3)^2 + (2)^2 + (1)^2}\sqrt{(2)^2 + (3)^2 + (2)^2}}$$

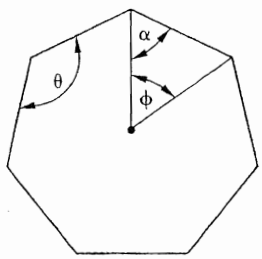
$$= \frac{14}{\sqrt{14}\sqrt{17}}$$

$$= 0.907485$$

$$\theta = \cos^{-1}(0.907485) = 24.8°$$

Answer is A.

14.



$$\phi = \frac{360°}{7} = 51.43°$$

$$\alpha = \frac{180° - \phi}{2} = 64.28°$$

$$\theta = 2\alpha = 128.6°$$

Answer is D.

15. $$\cos x = \sum_{n=0}^{\infty}\left(\frac{x^{2n}}{(2n)!}\right)(-1)^n$$

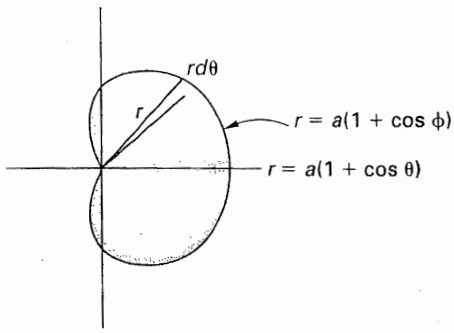
Answer is B.

16. **Answer is C.**

17. $$\overline{A} \cup B = (5, 6, 8, 10) \cup (4, 5, 9, 10)$$
$$= (4, 5, 6, 8, 9, 10)$$

Answer is C.

18. Graph the polar coordinate system.



By definition,

$$A = \tfrac{1}{2}\int_0^{2\pi}[f(\theta)]^2 d\theta$$

Since the curve is symmetrical,

$$A = \left(\tfrac{1}{2}\right)(2)\int_0^{\pi}[f(\theta)]^2 d\theta = a^2\int_0^{\pi}(1 + \cos\theta)^2 d\theta$$

$$= a^2\int_0^{\pi}(1 + 2\cos\theta + \cos^2\theta)d\theta$$

$$= a^2\left[\theta + 2\sin\theta + \tfrac{1}{2}\theta + \tfrac{1}{4}\sin 2\theta\right]_0^{\pi}$$

$$= a^2\left[\tfrac{3}{2}\theta + 2\sin\theta + \tfrac{1}{4}\sin 2\theta\right]_0^{\pi}$$

$$= a^2\left(\frac{3\pi}{2} + 0 + 0 - 0 - 0 - 0\right) = \frac{3\pi a^2}{2}$$

Answer is A.

19. Using matrix multiplication,

$$9B_1 + 7B_2 = 2$$
$$B_1 + 3B_2 = 1$$

Solving simultaneously,

$$-20B_2 = -7$$

$$B_2 = 7/20$$

$$B_1 = 1 - 3B_2 = 1 - (3)\left(\frac{7}{20}\right) = -1/20$$

Answer is A.

20. Divide through by x.

$$y' + 3 - \frac{1}{x} = 0$$

Simplify.

$$\frac{dy}{dx} = \frac{1}{x} - 3$$

$$dy = \left(\frac{1}{x} - 3\right) dx$$

Answer is D.

21.
$$x\,\frac{dy}{dx} + 3x - 1 = 0$$

$$dy = \left(\frac{1 - 3x}{x}\right) dx = \left(\frac{1}{x} - 3\right) dx$$

$$y = \ln x - 3x + C$$

Answer is C.

22. From Prob. 21,

$$y = \ln x - 3x + C$$

$$l = \ln(1) - (3)(1) + C$$

$$C = 4$$

Answer is D.

23. This is the vector triple scalar product, which is zero only if $\overline{A}$ lies in the same plane as $\overline{B}$ and $\overline{C}$.

Answer is B.

24. **A** is 2×3 and **B** is 2×2, so **BA** exists, but not **AB**.

Answer is A.

25. This is a current divider.

$$i_{6\,\Omega} = (6\text{ A})\left(\frac{18\ \Omega}{6\ \Omega + 18\ \Omega}\right) = 4.5\text{ A}$$

Answer is D.

26.
$$C_{eq} = 30\ \mu\text{F} + \frac{1}{\frac{1}{20\ \mu\text{F}} + \frac{1}{20\ \mu\text{F}}} = 40\ \mu\text{F}$$

Answer is C.

27.
$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(4\text{ H})(3 \times 10^{-6}\text{ F})}}$$

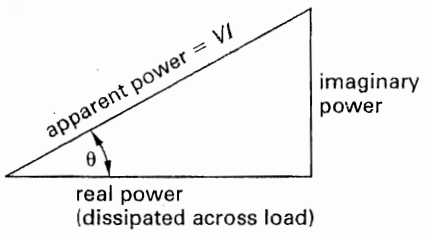
$$= 45.94\text{ Hz}$$

Answer is C.

28. Time-dependent terms will be strictly sinusoidal without decay.

Answer is A.

29.



$$\text{power factor} = \cos\theta = \frac{\text{real power}}{\text{apparent power}}$$

Answer is B.

30.
$$V_{line} = \sqrt{3}V_{phase}$$

$$V_{ab} = (\sqrt{3})(110\text{ V}) = 190.5\text{ V}$$

Answer is C.

31.
$$W = Fd = \frac{Vqd}{r}$$

$$\frac{V}{r} = 50\text{ V/m}$$

$$q = 10\text{ C}$$

$$d = 5\text{ m}$$

$$W = \left(50\ \frac{\text{W}}{\text{m}}\right)(10\text{ C})(5\text{ m}) = 2500\text{ W·C}$$

$$= 2500\text{ J}\quad (2.5\text{ kJ})$$

Answer is C.

32.
$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$\alpha = \frac{1}{2RC}$$

overdamped if $\alpha > \omega_o$

underdamped if $\omega_o > \alpha$

Answer is D.