Solutions for Exam 1–Morning Section

1.
$$\frac{N}{N_0} = \left(\tfrac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$
$$0.01 = \left(\tfrac{1}{2}\right)^{\frac{t}{4.3\text{ days}}}$$
$$\ln(0.01) = \left(\frac{t}{4.3\text{ days}}\right)\ln\left(\tfrac{1}{2}\right)$$
$$t = (4.3\text{ days})\left(\frac{\ln(0.01)}{\ln(0.5)}\right) = 28.6\text{ days}$$

Answer is C.

2.
$$\begin{aligned} p\{\text{at least 2}\} &= p\{2\text{ or }3\text{ or }4\text{ or }5\} \\ &= 1 - p\{0\text{ or }1\} \\ &= 1 - p\{0\} - p\{1\} \end{aligned}$$

Use binomial distribution to find the individual probabilities.

$$\begin{aligned} p &= \text{probability of heads} = 0.5 \\ q &= \text{probability of tails} = 0.5 \\ p\{0\} &= (0.5)^5 = 0.03125 \\ p\{1\} &= \binom{5}{1}(0.5)^4(0.5)^1 = 0.15625 \\ p\{\text{at least 2}\} &= 1 - 0.03125 - 0.15625 = 0.8125 \end{aligned}$$

Answer is C.

3.
$$(1)(2) - (4)(3) = -10$$

Answer is A.

4.
$$\begin{aligned} (x^2 - 6x + 9) + (y^2 - 4y + 4) - 12 &= 9 + 4 \\ (x-3)^2 + (y-2)^2 &= 25 \\ r &= \sqrt{25} = 5 \end{aligned}$$

Answer is C.

5. Solve by factoring.

$$\begin{aligned} \lim_{x\to 3}\frac{x^2-9}{x-3} &= \lim_{x\to 3}\frac{(x+3)(x-3)}{x-3} \\ &= \lim_{x\to 3}(x+3) \\ &= 6 \end{aligned}$$

Answer is C.

6. The magnitude of vector **V** is

$$V = \sqrt{(1)^2 + (2)^2 + (1)^2} = \sqrt{6}$$

The x-direction cosine is

$$\cos\phi_x = \frac{V_x}{V} = \frac{1}{\sqrt{6}}$$
$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{6}}\right) = 65.9°$$

Answer is C.

7.
$$\int_2^\infty \frac{1}{x^2}\,dx = -\frac{1}{x}\bigg|_2^\infty = \frac{-1}{\infty} - \frac{-1}{2}$$
$$= 1/2$$

Answer is C.

8.
$$\text{mean} = \frac{1+4+7}{3} = 4$$

The standard deviation is

$$\sigma = \sqrt{\frac{(1-4)^2 + (4-4)^2 + (7-4)^2}{3}} = \sqrt{6}$$
$$= 2.45$$

Notice that $n-1$ would have been used if the sample standard deviation had been requested.

Answer is A.

9. The magnitudes of the two vectors are

$$\begin{aligned} V_1 &= \sqrt{(1)^2 + (2)^2 + (1)^2} = \sqrt{6} \\ V_2 &= \sqrt{(1)^2 + (3)^2 + (-7)^2} = \sqrt{59} \\ \phi &= \cos^{-1}\left[\frac{(1)(1) + (2)(3) + (1)(-7)}{\left(\sqrt{6}\right)\left(\sqrt{59}\right)}\right] = 90° \end{aligned}$$

Answer is C.

10.
$$A = \int_0^1 e^x\,dx = e^x\bigg|_0^1 = e^1 - e^0$$
$$\approx 1.718$$

Answer is B.

11. The first and second derivatives are

$$y' = x^3 - 3x + 2$$
$$y'' = 3x^2 - 3$$

For a critical point, $y' = 0$. By inspection (based on the four answer choices), $y' = 0$ at $x = 1$ and $x = -2$.

$$y''(1) = (3)(1)^2 - 3 = 0$$
$$y''(-2) = (3)(-2)^2 - 3 = 9$$
$$y(-2) = \left(\tfrac{1}{4}\right)(-2)^4 - (1.5)(-2)^2 + (2)(-2) + 5 = -1$$

Answer is A.

12. $$\theta = \tan^{-1}\left(\tfrac{1}{2}\right) - \tan^{-1}\left(-\tfrac{2}{3}\right) = 60.26°$$

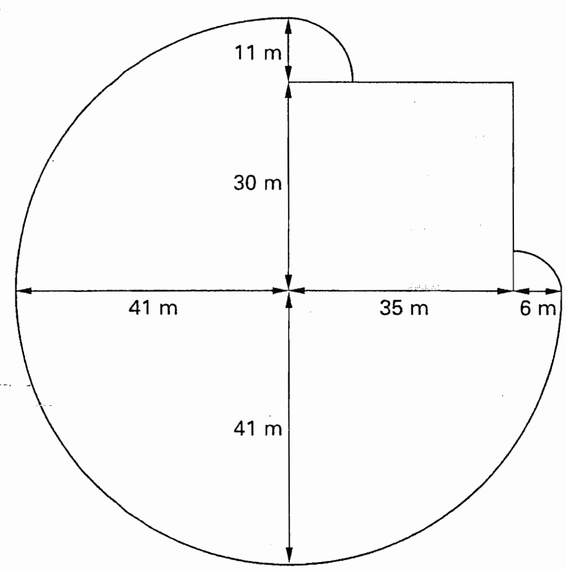
Answer is C.

13. $$\left(y^2 + y + \tfrac{1}{4}\right) + (x^2 - 2x + 1) = 5 + \tfrac{1}{4} + 1$$
$$= 25/4$$
$$\left(y + \tfrac{1}{2}\right)^2 + (x-1)^2 = 25/4$$

This is a circle centered at $(1, -1/2)$, so y_{max} is at $x = 1$.

Answer is C.

14.



$$\tfrac{3}{4}\pi(41\ \text{m})^2 + \tfrac{1}{4}\pi(11\ \text{m})^2 + \tfrac{1}{4}\pi(6\ \text{m})^2 = 4084.1\ \text{m}^2$$

Answer is D.

15. Differentiating,

$$y' = 27x^2 + 2x - 15$$
$$y'' = 54x + 2 = 0$$
$$x = -\frac{2}{54} = -0.037$$
$$y(-0.037) = 32.56$$
$$(-0.037, 32.56)$$

Answer is C.

16. $$|\mathbf{R}| = \sqrt{(1+2-1)^2 + (3+7+4)^2 + (4-1+2)^2}$$
$$= \sqrt{4 + 196 + 25}$$
$$= \sqrt{225}$$
$$= 15$$

Answer is C.

17. Multiplying through by 2 gives

$$x'' + 8x' + 16x = 10$$

The characteristic equation is

$$r^2 + 8r + 16 = 0$$

The roots of the characteristic equation are

$$r_1 = r_2 = -4$$

The homogeneous (natural) response is

$$x_{\text{natural}} = Ae^{-4t} + Bte^{-4t}$$

By inspection, $x = 5/8$ is a particular solution that solves the nonhomogeneous equation, so the total response is

$$x = Ae^{-4t} + Bte^{-4t} + \tfrac{5}{8}$$

Since $x = 1$ at $t = 0$,

$$1 = Ae^0 + \tfrac{5}{8}$$
$$A = \tfrac{3}{8}$$

Differentiating x,

$$x' = \left(\tfrac{3}{8}\right)(-4)e^{-4t} + B\left(-4te^{-4t} + e^{-4t}\right) + 0$$

Since $x' = 0$ at $t = 0$,

$$0 = -\tfrac{3}{2} + B(0 + 1)$$
$$B = \tfrac{3}{2}$$
$$x = \tfrac{3}{8}e^{-4t} + \tfrac{3}{2}te^{-4t} + \tfrac{5}{8}$$

Answer is D.

18. Since $\log_b b^{f(x)} = f(x)$,

$$\log_e e^{-7x} = -7x$$

Answer is C.

19. $$\text{probability} = \frac{\text{number of orange balls}}{\text{total number of balls}} = \frac{7}{17} = 0.4118$$

Answer is D.

20. Since $y'' = 14$, this is a maximum value.

$$y' = 14x - 3 = 0$$
$$x = 3/14$$
$$y\left(\frac{3}{14}\right) = (7)\left(\frac{3}{14}\right)^2 - (3)\left(\frac{3}{14}\right) + 8 = (7)\left(\frac{9}{196}\right) - \frac{9}{14} + 8 = 215/28$$

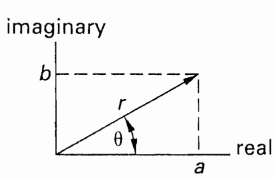
Answer is D.

21. From the double angle formulas,

$$\sin 2\theta = 2\sin\theta\cos\theta$$

Answer is A.

22.

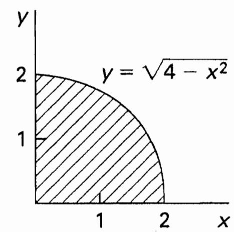


Use Euler's formula.

$$e^{\theta} = \cos\theta + i\sin\theta$$
$$\theta = \pi/2$$
$$e^{i\frac{\pi}{2}} = i$$
$$i^i = \left(e^{i\frac{\pi}{2}}\right)^i = e^{-\frac{\pi}{2}}$$

Answer is D.

23. A graph of the area is shown.



Since $y = \sqrt{4 - x^2}$ is the equation for the top half of a circle, the volume after a revolution will be a hemisphere.

$$V = \tfrac{1}{2}V_{\text{sphere}} = \left(\tfrac{1}{2}\right)\left(\tfrac{4}{3}\right)\pi r^3 = \left(\tfrac{1}{2}\right)\left(\tfrac{4}{3}\right)\pi(2)^3 = 16.76$$

Answer is C.

24. Cross products can only be performed with vectors.

Answer is D.

25. $$i = \frac{\Delta Q}{\Delta t} = \frac{6\ \text{C}}{2\ \text{s}} = 3\ \text{A}$$

Answer is B.

26. $$a = \frac{N_p}{N_s} = \frac{200\ \text{turns}}{20\ \text{turns}} = 10$$
$$V_s = \frac{V_p}{a} = \frac{120\ \text{V}}{10} = 12\ \text{V}$$

Answer is B.

27. $$\frac{N_p}{N_s} = a = \frac{I_s}{I_p} = \frac{0.3\ \text{A}}{30\ \text{A}} = 0.01\quad (1{:}100)$$

Answer is A.

28. $$P_p = I_p^2 R_p$$
$$I_p = \sqrt{\frac{P_p}{R_p}} = \sqrt{\frac{5\ \text{W}}{2000\ \Omega}} = 0.05\ \text{A}$$
$$I_s = aI_p = (15)(0.05\ \text{A}) = 0.75\ \text{A}$$
$$R_s = \frac{P_s}{I_s^2} = \frac{P_p}{I_s^2} = \frac{5\ \text{W}}{(0.75\ \text{A})^2} = 8.89\ \Omega$$

Answer is A.